647510 **BSCabsCC6020** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April -2020
MATHEMATICAL ANALYSIS-2 AND ABSTRACT ALGEBRA -2 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

1(A) Answer any two: **(10)**

(1) Prove that every compact set of a metric space is closed and bounded.

(2) State and prove necessary and sufficient condition for a subset of discrete metric space to be connected set.

(3) If A and B are compact sets of metric space X then show that $A \cup B$ and $A \cap B$ are also compact.

(B) Answer any one: **(04)**

(1) Define: Homeomorphism between two metric spaces.

If (X, d) and (Y, d') are two metric spaces, where

$X = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots \ldots.\right\}$and $Y = N$ and d, d' are standard

metric functions on X and Y respectively then prove that

$X$and Y are homomorphic to each other.

(2) Prove that the set of all real number R is not countable.

2 (A) Answer any two: **(10)**

(1) Find $L^{-1}\left\{\frac{s^3}{s^4 - a^4}\right\}$.

(2) State and prove the first shifting theorem of Laplace transforms and

find (i) $L[e^{-2t}(4cosh3t - 2sinht)]$(ii) $L[t^2 sinat]$.

(3) Define: Inverse Laplace transform.

Find: $L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$.

(B) Answer any one: **(04)**

(1) Find the Laplace transform of $f(t)$ defined as

(i) $f(t) = t, 0 < t < 4$ (ii) $f(t) = e^t, t \leq 2$

$5, t > 43, t > 2$

(2) Find: (i) $L^{-1}\left\{\frac{s+1}{s^2+s+1}\right\}$ (ii) $L^{-1}\left\{\frac{s+2}{(s-2)^3}\right\}$

3 (A) Answer any two: **(10)**

(1) Derive the formula for finding Laplace transform of the n^{th} derivative of $f(t)$. Using it find $L\{t^n\}$.

(2) If $L\{f(t)\} = \acute{f}(s)$ then prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\left[\acute{f}(s)\right]$, where n=1,2, 3,…

Also evaluate $L(t^2 sin4t)$.

(3) Using convolution theorem, find (i) $L^{-1}\left\{\frac{1}{(s+1)(s^2-2s+2)}\right\}$ (ii) $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$.

(B) Answer any one: **(04)**

(1) If $L\{f(t)\} = \acute{f}(s)$and $\frac{f(t)}{t}$ has Laplace transform then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_0^\infty f(s)ds.$

(2) Evaluate: (i) $L\left(\frac{cos2t-cos3t}{t}\right)$ (ii) $L^{-1}\left[log\left(\frac{s+b}{s+a}\right)\right]$.

4 (A) Answer any two: **(10)**

(1) Define: Kernel of a homomorphism. If φ: is homomorphism then prove that kernel K_φ is a normal subgroup of G.

(2) Prove that the commutative ring R with unity is a field if it has no proper ideal.

(3) If $\varphi: G \to G'$ is an onto homomorphism and K Is the kernel of φ then prove that ${}^{G}/_{k} \cong G'$.

(B) Answer any one: **(04)**

(1) Find all homomorphism from $\varphi: \mathbb{Z} \to \mathbb{Z}$ which are into. How many of these are onto?

(2) Define: Left and right ideal. Give an example of right ideal which is not left ideal.

5 (A) Answer any two: **(10)**

(1) For an integral domain D, prove that D[x] is also an integral domain with respect to the binary operations of addition and multiplication of polynomial.

(2) State and prove division algorithm for polynomials.

(3) Prove that any ideal in integral domain F[x] is a principal ideal.

(B) Answer any one: **(04)**

(1) Find the g.c.d. of $f(x) = x^3 + 3x^2 + 3x + 3$ and $g(x) = 4x^3 + 2x^2 + 2x + 2 \in Z_5[x]$and express it in the form $a(x)f(x) + b(x)g(x)$.

(2) Define: Inverse of quaternion. Simplify: (i) $i^2j^3kj^2i^3$ (ii) $(1+i)^{-1}(2j-3k)^{-1}$

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